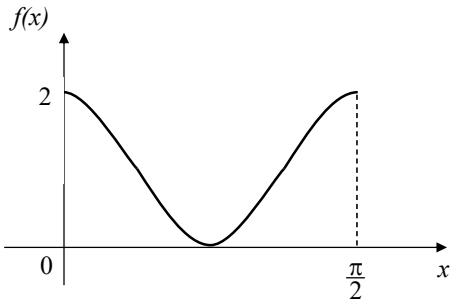
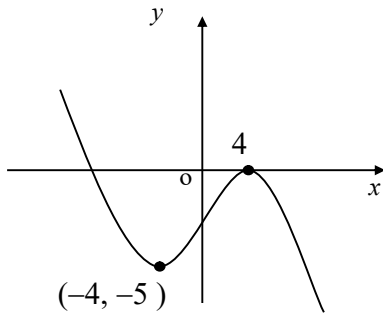


	Higher Prelim Revision 1 – Paper 1 Non-Calculator	30
1.	$x - 2$ is a factor of the polynomial $x^3 - 2x^2 + kx - 10$, find the value of k	2
2.	Two functions are defined on suitable domains as $f(x) = 2x - 4$ and $g(x) = x^2 + 1$. Find $f(g(3))$	3
3.	Given that the vectors $\begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} p \\ -2 \\ 3 \end{pmatrix}$ are perpendicular, find the value of p	2
4.	If the function $f(x) = 3x - 1$. Find the inverse function $f^{-1}(x)$	2
5.	The diagram shows part of the trigonometric function $y = \cos(ax) + b$ State the values of a and b	2
		
6.	Find the gradient of the tangent to the curve $y = \sin^2(x)$ When $x = \frac{\pi}{3}$	2
7.	The quadratic equation $4x^2 + kx + 1 = 0$ has real roots. State the range of values for k	3
8.	Find the rate of change of the function $f(x) = x^3 - 5x^2$ when $x = 5$	2

9. Part of the graph of the function $y = f(x)$ is shown below. The minimum turning point has coordinates $(-4, -5)$ and the maximum turning point has coordinates $(4, 0)$



Sketch:

- (a) $y = 2f(x)$
 (b) $y = 1 - 2f(x)$

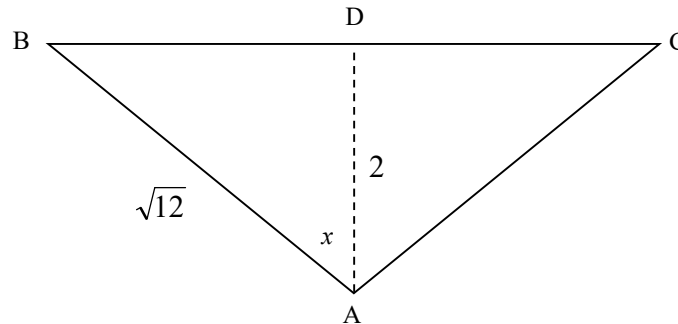
2

3

10. Solve the equation $4 \cos 2\theta = 6 \cos \theta + 1$, for $0 < \theta < 2\pi$.

6

11. In the diagram below, triangle ABC is isosceles with $AB = AC$. D is the mid-point of BC. $AB = \sqrt{12}$ units and $AD = 2$ units as shown. Angle $BAD = x$.



- (a) Show clearly that $\sin x = \frac{\sqrt{2}}{\sqrt{3}}$.

3

- (b) Hence show that $\sin BAC = \frac{2\sqrt{2}}{3}$.

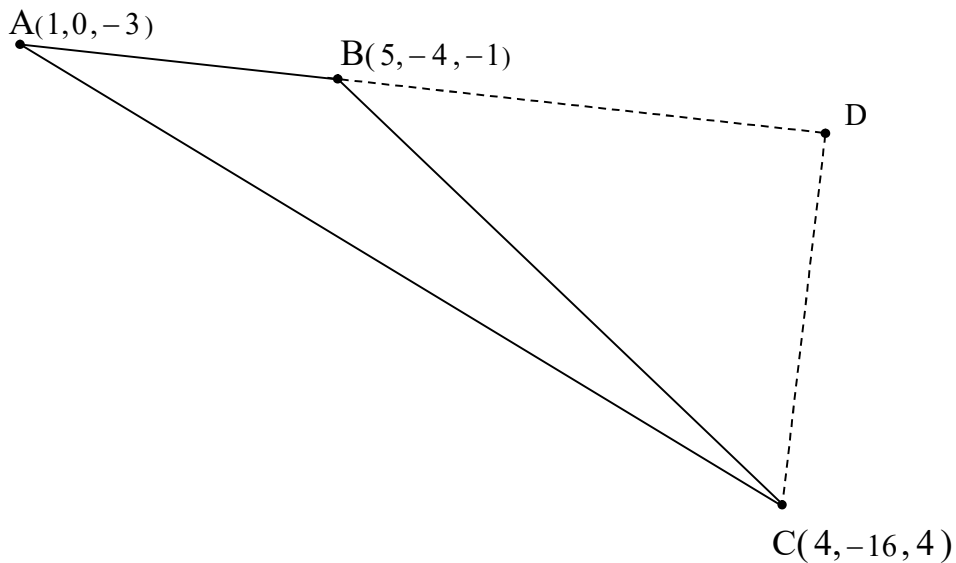
3

Answers to Paper 1

1. use synthetic division to find $2k = 0$,	$k = 5$
2. $f(g(x)) = f(x^2 + 1) = 2(x^2 + 1) - 4 = 2x^2 - 2$,	$f(g(3)) = 16$
3. $1 \times p + 4 \times -2 + 0 \times 3 = 0$, $p - 8 = 0$,	$p = 8$
4. $f(x) = 3x - 1 \rightarrow f(x) + 1 = 3x \rightarrow \frac{f(x) + 1}{3} = x \rightarrow$	$f^{-1}(x) = \frac{x + 1}{3}$
5. $y = \cos(4x) + 1$	$a = 4$, $b = 1$
6. Know to differentiate $y = \sin^2(x) \frac{dy}{dx} = 2 \sin(x) \cos(x)$	
Substitute $x = \frac{\pi}{3}$ into the derivative $2 \sin\left(\frac{\pi}{3}\right) \cos\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$	
7. for real roots $b^2 - 4ac \geq 0 \rightarrow a = 4$, $b = k$, $c = 1$	
$k^2 - 4(4)(1) \geq 0$	
$k^2 - 16 \geq 0$	
$(k - 4)(k + 4) \geq 0$	
$k \leq -4$ and $k \geq 4$	
8. Differentiate $f'(x) = 3x^2 - 10x \rightarrow f'(5) = 3(5)^2 - 10(5) = 25$	
9. (a) Vertical scaling by 2 $(-4, -5) \rightarrow (-4, -10)$ $(4, 0) \rightarrow (4, 0)$	
(b) Reflection in x-axis $(-4, -10) \rightarrow (-4, 10)$ $(4, 0) \rightarrow (4, 0)$	
Vertical translation +1 $(-4, 10) \rightarrow (-4, 11)$ $(4, 0) \rightarrow (4, 1)$	
Sketch should show both turning points and the origin clearly marked	
10. replace double angle $4(\cos^2 \theta - 1) = 6 \cos \theta + 1$	
Rearrange and factorise $(4 \cos \theta - 5)(2 \cos \theta + 1) = 0$,	
Solve $(4 \cos \theta - 5) = 0$ has no solutions	
$(2 \cos \theta + 1) = 0$ two solutions $\theta = \frac{2\pi}{3}$, $\theta = \frac{4\pi}{3}$	
11. (a) $BD = \sqrt{8}$, $\sin x = \frac{\sqrt{8}}{\sqrt{12}}$	(b) $\sin 2x = 2 \sin x \cos x$
Simplify this answer	$\cos x = \frac{2}{\sqrt{12}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$
	$\sin 2x = 2 \times \frac{\sqrt{2}}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{2\sqrt{2}}{3}$

1. Triangle ABC has vertices $A(1, 0, -3)$, $B(5, -4, -1)$ and $C(4, -16, 4)$

A, B and D are collinear such that $\frac{AB}{BD} = \frac{2}{3}$



- (a) Show that the coordinates of D are $(11, -10, 2)$. 2
- (b) Hence show clearly that angle ADC is a right angle. 4
- (c) Prove that angle ABC is obtuse. 3
2. (a) If $x - 1$ is a factor of $3x^3 + kx^2 + 4x - 13$, show that the value of k is 6. 3
- (b) Hence find the x -coordinate of the single stationary point on the curve with equation $y = 3x^3 + kx^2 + 4x - 13$ when k takes this value 4

3.	Express the function $g(x) = 3 \sin x + 2 \cos x$ in the form $k \cos(x-a)$, $0 \leq a \leq 360^\circ$ and state the maximum value of this function	5
4.	Solve $\log_3(x^2 - 4) - \log_3(x - 2) = 3$	4
5.	Evaluate $\int_0^2 (x^3 - 9x^2 + x) dx$	4
6.	A function f is given by $f(x) = (x^2 + 3)^{1/2}$. (a) Find $f'(x)$	3
	(b) Find algebraically the values of x for which $f'(x) = \frac{1}{2}$	3
7.	A function, defined on a suitable domain, has as its derivative $f'(x) = 3x^2 - \frac{10}{x^2}$. Given that $f(2) = 3$, find $f(x)$.	5

Answers to Paper 2

<p>1. $\vec{DA} = \begin{pmatrix} -10 \\ 10 \\ -5 \end{pmatrix}$ $\vec{DC} = \begin{pmatrix} -7 \\ -6 \\ 2 \end{pmatrix}$</p> <p>DA.DC = 70 - 60 - 10 = 0 since DA.DC = 0; $\angle ADC$ is right angled</p>	<p>$\vec{BA} = \begin{pmatrix} -4 \\ 4 \\ -2 \end{pmatrix}$ $\vec{BC} = \begin{pmatrix} -1 \\ -12 \\ 5 \end{pmatrix}$ BA.BC = -54</p> <p>Since $\cos\theta < 0$, θ is an obtuse angle No need to find the actual angle</p>
<p>2. (a) Use synthetic division to find $k - 6 = 0$; $k = 6$</p>	<p>(b) $\frac{dy}{dx} = 9x^2 + 12x + 4$ $9x^2 + 12x + 4 = 0$ at SP $(3x + 2)(3x + 2) = 0 \rightarrow x = -\frac{2}{3}$</p>
<p>3. $k \cos a = 2$, $k \sin a = 3$ $\sqrt{13} \cos(x - 56)^\circ$ max is $\sqrt{13}$</p>	
<p>4. $\log_3 \frac{x^2 - 4}{x - 2} = 3 \rightarrow \frac{x^2 - 4}{x - 2} = 27 \rightarrow \frac{(x - 2)(x + 2)}{x - 2} = 27 \rightarrow x + 2 = 27 \rightarrow x = 25$</p>	
<p>5. Integrate $\left[\frac{x^4}{4} - \frac{9x^3}{3} + \frac{x^2}{2} \right]_0^2$, substitute \rightarrow answer = -18</p>	
<p>6 (a) Differentiate $\frac{2x}{2(x^2 + 3)^{1/2}} \rightarrow \frac{x}{(x^2 + 3)^{1/2}} \rightarrow \frac{x}{\sqrt{x^2 + 3}}$</p> <p>b) solve $\frac{x}{\sqrt{x^2 + 3}} = \frac{1}{2} \rightarrow 2x = \sqrt{x^2 + 3} \rightarrow 4x^2 = x^2 + 3 \rightarrow 3x^2 = 3 \rightarrow x = \pm 1$</p>	
<p>7. Integrate $f(x) = \frac{3x^3}{3} - \frac{10x^{-1}}{-1} + C \rightarrow f(x) = x^3 + \frac{10}{x} + C$</p> <p>Solve for C $3 = 2^3 + \frac{10}{2} + C \rightarrow C = -10$, $f(x) = x^3 + \frac{10}{x} - 10$</p>	

Examples and Extra Practice

	Paper 1			Paper 2	
1.	Synthetic division	Ex 7.17 Pg 151 Q2,3 Pg 151	1.	Angles between Vectors	Ex 6.9 Pg 126 Q2,3 Pg 127
2.	Composite functions	Ex 4.3 Pg 86 Q1 Pg 87	2.	(a) Synthetic Division	Ex 7.14 Pg 147 Q1 Pg 149
3.	Perpendicular vectors	Ex 6.10 Pg 126 Q4,5 Pg 127	2.	(b) Stationary Points	Ex 10.10 Pg 253 Q1 Pg 256
4.	Inverse functions	Ex 4.6 Pg 89 Q1 Pg 89	3.	Wave Function	Ex 2.32 Pg 50 Q1 Pg 52
5.	Trig Graphs	Ex 3.23 Pg 75 Q5 Pg 77	4.	Log Equations	Ex 1.21 Pg 14 Q1 Pg 15
6.	Differentiation of trig functions	Ex 9.29 Pg 235 Q2, Pg 236	5.	Definite Integrals	Ex 12.1 Pg 285 Q1 Pg 286
7.	Discriminant	Ex 7.30 Pg 167 Q4 Pg 169	6.	Differentiation of composite functions	Ex 9.24 Pg 231 Q5 Pg 233
8.	Rate of Change	Ex 9.15 Pg 220 Q2 Pg 222	7.	Differential Equations	Ex 11.17 Pg 279 Q4,5 Pg 280
9.	Transposition of functions	Ex 3.2 Pg 56 Q1 Pg 58			
10.	Solving trig equations with double angles	Ex 8.14 Pg 188 Q2 Pg 191			
11.	Exact Values and Double Angles	Ex 2.18 Pg 38 Q2 Pg 40			